GENERALIZATION OF BASIC EQUATIONS OF AERODYNAMICS AND ELECTRODYNAMICS

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I WOULD like to make a report on preliminary results of my investigations of the following questions: (1) generalization of the equations of aerodynamics; (2) generalization of the equations of electrodynamics and (3) bringing them into one common system.

Because of the extraordinary extensiveness of the subject I have chosen, I have to confine myself here only to a report on the most essential things and pass over many important questions of principle which arise spontaneously in connection with my investigation.

The reference idea of the whole investigation is more than simple: the whole of physics including aerodynamics is based on experimental measurements, but the accuracy of our measurements is always limited; therefore all the rules both finite and in the form of differential equations which serve as a theoretical counterpart of experimental measurements, are always approximate; and almost all the rules of modern physics are only first approximations. No doubt, the system of the basic equations of electromagnetic field, derived 75 years ago by Maxwell, cannot envelop all the phenomena of electromagnetism known at the present time; today's theoretical physics tries to achieve this object with the help of superstructures in the form of relativistic, quantum and wave mechanics, changing, generalizing and even perverting the fundamentals of classical mechanics and physics, but admitting "tacito consensu" that the Maxwellian equations are absolutely correct. In my view, the Maxwellian equations of the electromagnetic field are only first approximations, and their inadequacy at the present time is due to the fact that the accuracy of measurements in electrodynamics has immeasurably increased compared with the time of Faraday, Maxwell and Hertz when they were originally derived. The same is true for the equations of hydrodynamics given by Euler 180 years ago and formally extended to the case of gas motion; they are obviously inadequate for the description of those rapid air flows which they have to deal with in aviation, especially for vortex flows. The Euler equations of aerodynamics are also only first approximations.

The things being as they are, it is quite natural to look for the second approximation both for equations of the electromagnetic field and aerodynamics without changing the fundamentals of classical mechanics and physics and to see whether these more general equations could cover the whole range of the facts in the field of electromagnetism and aerodynamics which have been exactly established by means of experiments.

Regarding the degree of approximation, we must establish a fixed criterion. It presents no difficulty to determine the latter, analyzing the circumstances under which the Euler and Maxwell equations were developed in their time. Thus for the criterion of approximation in aerodynamics we should take the ratio of the squared gas velocity to the squared sound

It is the ratio of the squared magnetic intensity M to the squared electric intensity, i.e. M^2/E^2, that serves as such a criterion in electrodynamics. This electrodynamic criterion absolutely coincides, as we shall see below, with the aerodynamic one, as the ratio of the magnetic intensity to the electric intensity can always be presented as the ratio of the electric field field velocity (w) to the light velocity:

$$\frac{M^2}{E^2} = \frac{w^2}{c^2}.$$

In the first approximation this ratio of the squared velocities is considered to be a very small quantity compared to unity and it may be neglected in equations or, if retained, taken as a correction factor. In the second approximation this ratio w^2/c^2 can be not only equal to unity but even greater. The Newtonian mechanics knows no restrictions concerning it, and therefore this quantity should be given a special attention in the second approximation.

When trying to find the second approximations, we should bear in mind not only that it deals with velocities equal and even higher than those of sound and, accordingly, light, but also the fact that after Euler's and Maxwell's time, there were established experimental facts which change radically the very interpretation of these problems. I mean the fact of discontinuity of the structures both of gas and electric field. In Euler's and Maxwell's derivation, on the contrary, continuity was postulated in these cases. Discontinuity of gas structure is established primarily by the kinetic gas theory and its agreement with facts, discontinuity of the electric field, by the existence of an elementary electric charge (electron).

For the basic method, when solving the problems set forth, I make use of the Lagrange equations of dynamics extended to physical systems by Helmholtz and given in his last works on the principle of least action. If Π denotes the potential energy of the system and K its kinetic energy, so that the kinetic potential will be $H = \Pi - K$, the Helmholtz equation for the parameter q_a and velocity $\dot{q}_a$ takes the form:

$$\frac{\partial H}{\partial q_a} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial H}{\partial \dot{q}_a} = 0. \tag{1}$$

Additivity of the expression for kinetic potential (H) is made on the basis of the experimentally established and generally recognized basic properties of the aerodynamic and electric fields.

Putting aside for the present all the necessary calculations that are of course complicated and demand much space, I shall consider only the main results of my investigation making only some applications for comparison with experiment in a number of special cases.

To make it more vivid and for a better comparison of the first and second approximations for the equations of both aerodynamics and electrodynamics, I shall make use of the wonderful method of ancient mathematicians, namely, I shall make use of the natural curvilinear orthogonal coordinates. To represent the Euler equations of aerodynamics in these coordinates, let us assume that the first axis λ of curvilinear coordinates is directed along the vortex axis, the third, axis ν is in the velocity direction w along the normal to the vortex axis and axis μ is perpendicular to λ and ν making with it the right-hand system of the coordinates; the velocity (v) will then be represented by two components: $u = h_1\lambda$ along the vortex axis (the longitudinal velocity) and $w = h_3\nu$ along the normal to the vortex (normal velocity), where $1/h_1$, $1/h_2$, $1/h_3$ are the Lame coefficients.

In the so chosen coordinates the Euler equations will be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho\tau) = 0 \tag{2}$$

$$\left.\begin{aligned} &\frac{1}{\rho}\frac{\partial p}{\partial \lambda} - \frac{\delta}{\delta\lambda}\left(\frac{u^2 + w^2}{2}\right) + \frac{\mathrm{d}}{\mathrm{d}t}(h_1 u) = 0 \\ &\frac{1}{\rho}\frac{\partial p}{\partial \mu} - \frac{\delta}{\delta\mu}\left(\frac{u^2 + w^2}{2}\right) = 0 \\ &\frac{1}{\rho}\frac{\partial}{\partial \nu} - \frac{\delta}{\delta v}\left(\frac{u^2 + w^2}{2}\right) + \frac{\mathrm{d}}{\mathrm{d}t}(h_3 w) = 0 \end{aligned}\right\} \tag{3}$$

Here τ is the particle volume, ρ is the density, p is the measure of gas elasticity; the external force effect is assumed to be zero; operations

$\delta/\delta\lambda, \ldots$ imply that these derivatives are taken on the assumption that λ, $\dot{v}$ do not change.

In the case of eddy-free motion it is necessary to add to these equations

$$\operatorname{curl} V = 0.$$

As the calculations show, the second approximation for eddy-free motion differs in its form from the first one very little, namely, the equations of the second approximation look as

$$\left.\begin{aligned}
&\frac{1}{\rho}\frac{\partial p}{\partial \lambda} - \frac{\partial}{\partial \lambda}\left(\frac{u^2 + w^2}{2}\right) + \frac{\mathrm{d}}{\mathrm{d}t}(h_1 u) = 0\\
&\frac{1}{\rho}\frac{\partial p}{\partial \mu} - \frac{\partial}{\partial \mu}\left(\frac{u^2 + w^2}{2}\right) = 0\\
&\frac{1}{\rho}\frac{\partial p}{\partial v} - \frac{\partial}{\partial v}\left(\frac{u^2 + w^2}{2}\right) + \frac{\mathrm{d}}{\mathrm{d}t}(h_3 w) = 0.
\end{aligned}\right\} \quad (4)$$

The change concerns only operations $\partial/\partial\lambda, \ldots$, which are here represented in their usual meaning.

This difference between equations (4) and (3) is the direct consequence of the discontinuity of gas structure.

Equations (4) can also be presented in such a form which permits us see better the degree of their approximation:

$$\left.\begin{aligned}
&\frac{\partial}{\partial \lambda}\left\{\left(1 - \frac{k-1}{2}\cdot\frac{u^2 + w^2}{c^2}\right)\cdot\int\frac{\mathrm{d}p}{\rho}\right\}\\
&\qquad + \frac{\mathrm{d}}{\mathrm{d}t}(h_1 u) = 0\ ;\\
&\frac{\partial}{\partial v}\left\{\left(1 - \frac{k-1}{2}\cdot\frac{u^2 + w^2}{c^2}\right)\cdot\int\frac{\mathrm{d}p}{\rho}\right\}\\
&\qquad + \frac{\mathrm{d}}{\mathrm{d}t}(h_3 w) = 0\ ;
\end{aligned}\right\} \quad (4^*)$$

where C^2 is the squared sound velocity and k is the adiabatic coefficient.

It is interesting to follow the difference between the solutions obtained by these two approximations in a p a r t i c u l a r case of motion.

Let us examine, for example, eddy-free motion of a gas round a cylindrical vortex. As is known, in the case of the Euler equations we have a paradoxical result, namely, the kinetic energy of this eddy-free motion based on the vortex unit length, tends to logarithmic infinity, and the gas density increases from the vortex surface towards the periphery. In other words, from this we must come to the conclusion that according to Euler, it is impossible to realize vortex motion in such a form. Meanwhile in experiment there is nothing simpler than getting a vortex column between two parallel planes with the two ends of the vortex rested on them, and air density evidently decreases in the direction from the vortex axis towards the free atmosphere. The second approximation yields that the gas density decreases from the centre towards the periphery in inverse proportion to $r^{2/k-1}$ and kinetic energy of the gas rotating round the vortex is finite and inversely proportional to the vortex diameter in power $(2/k - 1)$.

Thus an agreement with experiment belongs to the second approximation.

In the case of vortex motion the difference between the Euler equations and the second approximation is more considerable and our concepts of vortex motions which were found on the basis of Helmholtz' vortex theory are undergoing essential changes.

In the first approximation, in the case of vortex motion, the following condition is added to the Euler equations (2) and (3): $\operatorname{curl} V = 2\Omega_\lambda$, where Ω_λ is the angular vortex velocity or in an expanded form:

$$2\Omega_\lambda = \frac{1}{h_2 h_3}\frac{\partial}{\partial \mu}(h_3 w)\ ;\ 0 = \frac{\partial}{\partial v}(h_1 u) - \frac{\partial}{\partial \lambda}(h_3 w)\ ;$$

$$0 = \frac{\partial}{\partial \lambda}(h_1 u)\,.$$

Hence from this condition and equation (2) and (3) follow first, the continuity equation for the vortex over its whole length

$$\frac{\partial}{\partial \lambda}(\Omega_\lambda \sigma_\lambda) = 0 \qquad (5a)$$

and, second, the Helmholtz equation of vortex tension conservation:

$$\frac{d}{dt}(\Omega_\lambda \sigma_\lambda) = 0. \tag{5b}$$

Here σ_λ is the vortex cross-sectional area: $(\sigma_\lambda = h_2 h_3 d\mu dv)$.

Therefore, Helmholtz' theory of vortices is based on the Euler equations.

In the second approximation the theory of the vortex motion results directly from the Helmholtz equation for the kinetic potential and may be obtained in an absolutely exact form from the equation:

$$\frac{d}{dt}\left(\frac{\partial H}{\partial \Omega_\lambda}\right) = 0 \quad \text{or} \quad \frac{d}{dt}[(\Omega_\lambda \sigma_\lambda) . f(\gamma_k)] = 0. \tag{6}$$

Of course, the vortex continuity condition (5a) serves as a vortex characteristic.

In equation (6) the function $f(\gamma_k)$ has a rather complicated form, and

$$\gamma_k = (\Omega_\lambda \sigma_\lambda) . \Omega_\lambda^{(2/k-1)}.$$

Thus, equation (6) shows that the conservation of vortex tension ($\gamma = \Omega_\lambda \sigma_\lambda$) does not take place at all. Direct observations, of course, quite agree with this conclusion: a vortex is a rather fleeting movement, vortex motions easily appear and as easily disappear. Hence, basing our views on Helmholtz' theory, we attribute this property of vortices to gas viscosity (it should be noted that in the second approximation under consideration we, for the present, have passed the gas viscosity over).

But our observations of vortices in the atmosphere reveal the possibility of existing of vortices also in the form of storms, that are not destroyed by the obstacles they meet on their way, but they themselves destroy the obstacles. It means that such vortices are strong and steady.

Equation (6) gives also the opportunity to explain the existence of storms (water-spouts and sand storms). If we integrate equation (6) with respect to time, we shall get vortex tension γ as a function of its angular velocity. It appears that γ has its maximum at a certain angular velocity, therefore close to this maximum the vortex is most steady; the value of the maximum is determined by three physical parameters, such as: adiabatic coefficient k, sound velocity c and the Avogadro number. For air we obtain $\gamma_{max} = 6 \cdot 10^7$ cm^2/s. On this ground the vortex cross-section will be determined because, as it will be seen below, linear velocity at the vortex periphery is equal to the sound velocity: the vortex cross-section is equal to 10 m and, accordingly, the angular velocity turns out to be of the order of 10 r.p.s. Everyone who watched storms in their most destructive phase estimates their cross sections to be just 10 m.

Thus the second approximation permits us to explain one of the most mysterious atmospheric phenomena.

Equation (6) leads us to even more important conclusions of general significance, that is, it appears that continuity equation (2) for the vortex case splits up into two equations:

$$\frac{d}{dt}(\rho \sigma_\lambda) = 0; \frac{d}{dt}(h_1 d\lambda) = 0. \tag{7}$$

These equations (7) imply that the length of any vortex consists of a number of sections, the length of every section and its mass remaining constant during the whole period of its motion.

It seems to me that we can check to some extent this very important consequence of equation (6) by such mental experiment: let us imagine a uniform vortex field between two parallel planes on which the ends of the vortices are resting. Suppose, we shall move these planes apart with the help of an external force, stretching the vortices. This will not affect the uniformity of the vortex field and its tension due to the symmetry of circumstances; the vortex field energy will increase due to external forces work, but at the same time the mass of the eddied gas will also increase, thus new vortices will be produced and the number of vortex sections will increase at the expense of the eddy coming from the outside of the eddy-free gas. Helmholtz' theory gives no possibility to explain such experiment.

For the case of a vortex the experiment described is only imaginable, but in a similar case with electrostatic field between the plates of a flat capacitor, it can be easily realized. And in fact, such a process of the field stretching makes the basis of frictionless electrostatic machines.

Equation (6) leads us to one more unexpected result, namely: from the expression for γ_k it is clear that for $k = 2$ it takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Omega_\lambda \sigma_\lambda) = 0 .$$

It means that only in this case we have conservation of the vortex tension; the vortex is quite steady.

As among the gases known, there is no gas with adiabatic constant equal to 2 (for all of them $k < 2$), this exclusive gas will be referred to as supergas.

So, the general conclusion from equation (6) can be formulated in the following way: for all real gases ($k < 2$) vortex motion is unsteady; once produced, the vortex immediately disappears, although under certain conditions vortex motion is possible under the action of external forces. Only for a supergas ($k = 2$) the vortex motion is quite steady. When setting up the equation of motion for a vortex system in constructing further the second approximation, we shall mean just this latter case and we shall suppose that

$$\frac{\gamma\Omega}{c^2} \ll 1$$

so, expanding the kinetic potential H into series, we can omit all the terms of the highest order of smallness, then the differential equations of the vortex system motion acquire the following comparatively simple form, quite different from the form the Euler equations give us: the first group of equations:

equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h_1 u) = 0 \tag{8}$$

$$\left.\begin{aligned} &\frac{1}{h_1}\frac{\partial}{\partial \mu}\left\{h_1\left(1 + \frac{u^2}{2c_L^2} - \frac{w^2}{2c_L^2}\right)\cdot \int \frac{\mathrm{d}p}{\rho}\right\} = 0 ; \\ &\frac{1}{h_1}\frac{\partial}{\partial \nu}\left\{h_1\left(1 + \frac{u^2}{2c_L^2} - \frac{w^2}{2c_L^2}\right)\cdot \int \frac{\mathrm{d}p}{\rho}\right\} + \frac{\mathrm{d}}{\mathrm{d}t}(h_3 w) = 0 ; \end{aligned}\right\} \tag{9}$$

and then the second group containing the angular vortex velocity

$$\frac{\partial}{\partial \mu}\left(h_1 h_3 \frac{w\Omega_\lambda}{c_L^2}\right) = 0 \quad \text{and} \quad \frac{\partial}{\partial \mu}(h_1 h_3^2) = 0 \tag{10}$$

$$\left.\begin{aligned} &\frac{1}{h_2 h_1}\frac{\partial}{\partial \mu}\left\{h_1 \Omega_\lambda \left(1 + \frac{u^2}{c_L^2} - \frac{w^2}{c_L^2}\right)\right\} = 0 \\ &\frac{1}{h_3 h_1}\frac{\nu}{\partial \nu}\left\{h_1 \Omega_\lambda \left(1 + \frac{u^2}{c_L^2} - \frac{w^2}{c_L^2}\right)\right\} \\ &\frac{1}{\sigma_\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\sigma_\mu \cdot \frac{w\Omega_\lambda}{c_L^2}\right) = 0 \\ &(\sigma_\mu = h_1 h_3 \mathrm{d}\lambda \mathrm{d}\nu) \end{aligned}\right\} \tag{11}$$

For $k < 2$, for real gases, instead of 1 and 2 we must substitute $1/(k-1)$ and $2/(k-1)$, respectively, and instead of $c_L \sim 2c_N$, where c_L is the velocity of the sound, computed according to the Laplacian formula, and c_N to Newton's formula.

For the boundary conditions of the vortices system with eddy-free gas, we get from the same basic equation of Helmholtz for kinetic potential (H), and u, w being continuous, at the vortex ends:

$$\rho \cdot \left\{2 - \frac{u^2 + w^2}{c^2}\right\} = 0 \quad \text{and} \quad u = 0 \,. \quad (12)$$

At the side boundary

$$1 + \frac{u^2}{c^2} - \frac{w^2}{c^2} = 0 \,, \quad (13)$$

$$\int \frac{\mathrm{d}p'}{\rho^1} - w^2 = 0 \,, \quad (14)$$

where p^1 and ρ^1 are the elasticity and density of eddy-free gas.

Equations of Maxwell–Heavyside–Hertz for (8)—(14) clearly exhibit anisotropy of eddied gas properties, whereas in the Euler equations it is not expressed at all.

The longitudinal velocity being absent ($u = 0$), boundary condition (13) shows that at the periphery of an individual vortex the velocity should be equal to the sound velocity; we used this vortex property in two particular cases considered above.

We shall pass over to the second problem solved—to the generalization of electromagnetic field equations.

Equations of Maxwell–Heavyside–Hertz for "vacuum" in the vector form can be written as

$$\operatorname{div}(D_0 E) = 0 \quad \text{(a)}; \operatorname{curl} M = \frac{D_0}{C_0}\frac{\partial E}{\partial t} \quad \text{(b)}; \; (15)$$

$$\operatorname{div}(\mu_0 M) = 0 \quad \text{(a)}; \operatorname{curl} E = -\frac{\mu_0}{C_0}\frac{\partial M}{\partial t} \, \text{(b)}. \; (16)$$

where $D_0\mu_0 = 1$, D_0 is the dielectric constant of "vacuum", which is ordinarily assumed to be unity, C_0 is the light velocity in "vacuum", E and M are the intensities of electric and magnetic fields.

Using here also natural curvilinear orthogonal coordinates λ, μ, ν, axis λ being directed along E, axis μ being directed along M and ν being orthogonal to them and directed along the normal velocity of electric field w, we are able to write down the equations of the electric field in a more symmetrical and, what is more important, in a more obvious form:

$$\frac{\partial}{\partial \lambda}(D_0 E \sigma_\lambda) = 0 \quad \text{(a)}; \frac{\partial}{\partial t}(D_0 E) + \frac{1}{h_2 h_3}\frac{\partial}{\partial \nu}(h_2 c_0 M) = 0 \quad \text{(b)}; \quad (15^*)$$

$$\left.\begin{aligned} &\frac{\partial}{\partial \mu}(\mu_0 M \sigma_\mu) = 0 \\ &\text{(a)}; \frac{1}{h_1 h_2}\frac{\partial}{\partial \mu}(h_1 D_0 E) = 0 \,; \\ &\frac{1}{h_3 h_1}\frac{\partial}{\partial \nu}(h_1 D_0 E) = -\frac{\partial}{\partial t}\left(\frac{M}{c_0}\right) \quad \text{(b)}. \end{aligned}\right\} (16^*)$$

From equation of the continuity of the electric field intensity (15*, a) and owing to the existence of minimum elementary charge ε, we get in ordinary units:

$$D_0 E \sigma_\lambda = 4\pi\varepsilon \,. \quad (17)$$

As the elementary charge is conserved, we must have

$$\frac{\mathrm{d}}{\mathrm{d}t}(D_0 E \sigma_\lambda) = 0 \,,$$

or in detail:

$$\frac{\partial}{\partial t}(D_0 E) + \frac{1}{h_2 h_3}\frac{\partial}{\partial \nu}(h_2 w D_0 E) = 0 \,. \quad (18)$$

Comparing (18) with (15*, b), we get G. G. Thomson's relationship found in 1891, between the magnetic intensity M, electric intensity E the normal velocity w:

$$c_0 M = w D_0 E \,. \quad (19)$$

Using this relationship, we can eliminate the magnetic intensity from equation (16*)

$$\frac{\partial}{\partial \mu}\left(\mu_0 \frac{w D_0 E}{c_0}\sigma_\mu\right) = 0 \quad \text{(a)};$$

$$\frac{1}{h_3 h_1}\frac{\partial}{\partial \nu}(h_1 D_0 E) = -\frac{\partial}{\partial t}\left(\frac{w D_0 E}{c_0^2}\right) \quad \text{(b)}. \quad (16^{**})$$

The Maxwell equations being represented in such a form, it will be easier for us to compare them with the second approximation for electromagnetic field equations.

The second approximation is easily got from the Helmholtz general equation (1) for the kinetic potential, taking into consideration Thomson's relationship and usual expressions for electric and magnetic energies; equations of the electric field continuity (15*, a) and of charge conservation (18) represent a characteristic property of the electromagnetic field, and they are, of course, maintained in the second approximation. For the equations of the electric field motion in the second aproximation we get the following system of equations:

$$\left.\begin{aligned} &\frac{1}{h_1 h_2}\frac{\partial}{\partial\mu}\left\{h_1 D_0 E\left(1+\frac{u^2}{c_0^2}-\frac{w^2}{c_0^2}\right)\right\}=0\,;\\ &\frac{1}{h_3 h_1}\frac{\partial}{\partial v}\left\{h_1 D_0 E\left(1+\frac{u^2}{c_0^2}-\frac{w^2}{c_0^2}\right)\right\}=\\ &\qquad -\frac{1}{c_\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{wD_0E}{c_0^2}\sigma_\mu\right). \end{aligned}\right\} \quad (20)$$

In an expanded form

$$\frac{1}{c_\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{wD_0E}{c_0^2}\sigma_\mu\right)=\frac{\partial}{\partial t}\left(\frac{wD_0E}{c_0^2}\right)+\frac{1}{h_1h_3}\frac{\partial}{\partial v}\left(h_1\frac{w^2D_0E}{c_0^2}\right)+\frac{1}{h_2h_1}\frac{\partial}{\partial\lambda}\left(h_3\frac{uw}{c_0^2}D_0E\right);$$

for $u = 0$, the last term falls out and then equations (20) and (16**, b) quite coincide, i.e. in this case we get the Maxwell equations.

As formula (20) shows, the difference between the second approximation and the first one is that in the left-hand side of the last system of equations factor $g(c_0^2) = [1 + (u^2/c_0^2) - (w^2/c_0^2)]$ appears and the right-hand side involves the total time derivative of the magnetic intensity flux instead of the local time derivative from magnetic field intensity $\mathbf{M}$:

$$\frac{1}{D_0}\cdot\frac{wD_0E}{c_0}\sigma_\mu=\mu_0M\sigma_\mu\,.$$

Comparing these equations for the electromagnetic field with those for the vortex field in the case of supergas (in the second approximation) derived earlier, we shall see the full parallelism of these equations. For a better comparison we may omit constant values and then we get:

vortex field equations

$$\frac{\partial}{\partial\lambda}(\Omega_\lambda\sigma_\lambda)=0\,;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Omega_\lambda\sigma_\lambda)=0\,;$$

$$\frac{\partial}{\partial\mu}\left(\frac{w\Omega_\lambda}{c^2}\sigma_\mu\right)=0\,;$$

$$\frac{1}{h_1h_2}\frac{\partial}{\partial\mu}\{h_1\Omega_\lambda\,.\,g(c^2)\}=0\,;$$

$$\frac{1}{h_1h_3}\frac{\partial}{\partial v}\{h_1\Omega_\lambda\,.\,g(c^2)\}+\frac{1}{\sigma_\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{w\Omega_\lambda}{c^2}\sigma_\mu\right)=0\,.$$

electromagnetic field equations

$$\frac{\partial}{\partial\lambda}(E\sigma_\lambda)=0\,;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(E\sigma_\lambda)=0\,;$$

$$\frac{\partial}{\partial\mu}\left(\frac{wE}{c_0^2}\sigma_\mu\right)=0\,;$$

$$\frac{1}{h_1h_2}\frac{\partial}{\partial\mu}\{h_1E\,.\,g(c_0^2)\}=0\,;$$

$$\frac{1}{h_1h_3}\frac{\partial}{\partial v}\{h_1E\,.\,g(c_0^2)\}+\frac{1}{\sigma_\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{wE}{c_0^2}\sigma_\mu\right)=0.$$

From this comparison we see that Ω and E are proportional values and c_0^2 for the electromagnetic field corresponds to c^2 for the vortex one. We shall obtain the proportionality factor value by equating the energy expressions for a unit volume in the electromagnetic field and in the vortex one and assuming that for supergas the "sound" velocity is equal to the light velocity. As a result, we have for the elementary electric charge:

$$\varepsilon = \sqrt{\left[\frac{c_0^2}{2\pi\rho_0}\cdot(\rho\sigma_\lambda)\right]},$$

i.e. the elementary electric charge is proportional to the mass distributed over the elementary vortex cross-section. It is for the first time that the theory manages to "materialize" an electric charge, but at the same time it is evident that the very notion of "charge" loses its former meaning and may be used only as a measure of elementary "electric induction flux" (17).

When passing to the last part of my report, first of all I should like to note that the just considered equation system for the electromagnetic field in the second approximation is insufficient for the complete determination of the field velocity v and that c_0^2 is constant, whereas c^2 corresponding to it for the vortex field is a function of time and coordinates. I think, the way out of this difficulty consists in taking all the equations of the vortex field as the generalized equations of the electromagnetic one. Then expressing the former in "electromagnetic terms", we can express these generalized equations of the electromagnetic field in the following finite form.

The main characteristic of the field:

$$\frac{\partial}{\partial\lambda}(E_\lambda^*\sigma_\lambda) = 0 \quad (a); \quad \frac{d}{dt}(E_\lambda^*\sigma_\lambda) = \quad (b)$$

$$E_\lambda^*\sigma_\lambda = 4\pi\varepsilon, \qquad (I)$$

where

$$E^* = D_0 E.$$

And further, the binding equations and those of the field motion

$$\left.\begin{aligned} &\frac{\partial}{\partial\mu}(\mu^* M_\mu\sigma_\mu) = 0 \qquad (a);\\ &\frac{1}{h_1 h_2}\frac{\partial}{\partial\mu}\left\{h_1 E_\lambda\left(1+\beta^2-2\frac{M_\mu^2}{E_\lambda^2}\right)\right\} = 0 \quad (b);\\ &\frac{1}{h_3 h_1}\frac{\partial}{\partial\nu}\left\{h_1 E_\lambda\left(1+\beta^2-2\frac{M_\mu^2}{E_\lambda^2}\right)\right\} = \\ &\qquad\qquad \frac{1}{c_0\sigma_\mu}\frac{d}{dt}(\mu^* M_\mu\sigma_\mu) \end{aligned}\right\}(II)$$

where

$$\mu^* = \frac{c_0}{cD_0}; \beta^2 = \frac{u^2+w^2}{c^2} = \frac{v^2}{c^2};$$

and finally the continuity equations and the equations determining c^2

$$\left.\begin{aligned} &\frac{d}{dt}(c^2\tau) = 0; \quad \frac{d}{dt}(h_1 d\lambda) = 0;\\ &\qquad\qquad \frac{d}{dt}(h_1 u) = 0;\\ &\frac{1}{h_1}\frac{\partial}{\partial\mu}\left\{h_1\left(c^2+\frac{u^2}{2}-\frac{w^2}{2}\right)\right\} = 0;\\ &\frac{1}{h_1}\frac{\partial}{\partial\nu}\left\{h_1\left(c^2+\frac{u^2}{2}-\frac{w^2}{2}\right)\right\}\\ &\qquad\qquad + \frac{d}{dt}(h_3 w) = 0. \end{aligned}\right\}(III)$$

The last system of equations is only a transformed system of equations (2). (8) and (9), as for supergas $\int dp/\rho$ is equal to c^2.

Thus the generalized equations of the electromagnetic field differ from the Maxwell equations not only by the fact that they are non-linear, but also by the fact that the light velocity inside the field is variable.

As the equations are non-linear, when solving some particular problem we are sure to come to the solution of at least squared equation for the field components. Due to this fact it is obvious that equations (II) are able to give us the shape of the field only in a limited part of space, as for the remaining space the components will be imaginary values. But from the general expression for kinetic potential H for motion of the vortex system it turns out that there exist not only those equations represented by formulae (II), but that there is quite a system of differential equations of the same kind with only one change concerning

$$g(c^2) = \left(1 + \beta^2 - 2\frac{M^2}{E^{*2}}\right)$$

and just in this very expression we may substitute for unity any integer (but not too great), positive or negative, including zero. And the equations thus obtained will hold. As a result, such a system of differential equations with different

$$g_n(c^2) = \left(n + \beta^2 - 2\frac{M^2}{E^{*2}}\right)$$

determines the electromagnetic field in the whole space occupied by the system of vortices or by the electromagnetic field.

In this property of kinetic potential H and of the system of differential equations determining the field, I see the basis for explaining the field quantum properties. This very property of H is specified by discontinuous structure of the electric field; we may represent the electromagnetic field as a system of moving "Faraday tubes" similarly to the vortex field which we represent as a system of "vortex tubes". Equations of Faraday tubes movement and their properties are obtained from generalized equations (I), (II), (III) of the electromagnetic field, and these are quite similar to those of elementary vortex tubes motion. Mass motion and its distribution inside a vortex or Faraday tube are not expressed by these equations; to determine them, it is necessary to apply another method—statistic mechanics or kinetic theory of gases.

Having come to the conclusion of my report and restricting myself only to the details which are necessary for understanding of the solution procedure, I shall discuss a particular case when a system of vortices or Faraday tubes is reduced only to one tube, that is, the case of movement of an isolated vortex or Faraday tube. The interest in such an investigation lies in the field of electromagnetism—thus we may reveal the field structure of the main constituents of the electromagnetic field such as electron (positron), proton (anti-proton), neutron and proton.

I shall write the equations in the electromagnetic form, but describing the obtained solution properties I shall also use terms related to the vortex field because the solutions are valid in both of them.

For an isolated vortex or Faraday tube the basic differential equations are considerably simplified:

$$\operatorname{div} V = 0\,; \tag{1}$$

$$\operatorname{div} E^* = 0\,; \tag{2}$$

$$1 + \beta^2 - 2\frac{M^2}{E^{*2}} = 0\,. \tag{3}$$

Let us discuss the case when the Faraday tube moves with constant translational velocity $\beta_0 c_0$ and rotates also with constant angular velocity w round the axis directed along the translational velocity. Then the first equation will be satisfied. As for the second equation we may present it in spherical coordinates r, φ, ϑ:

$$\frac{\partial}{\partial r}(r^2 \sin\varphi E_r^*) + \frac{\partial}{\partial \varphi}(r \sin\varphi E_\varphi^*) + \frac{\partial}{\partial \varphi}(r E_\vartheta^*) = 0\,; \tag{2'}$$

$$1 + \beta_0^{*2} + \frac{\omega^2 r^2 \sin^2\varphi}{c^2} - 2\frac{M^2}{E^{*2}} = 0\,. \tag{3'}$$

where

$$\beta_0^* = \beta_0 \cdot \frac{c_0}{c}\,.$$

Besides, we also have an equation of kinematic nature which implies that for an observer who rotates together with the field, the latter will remain unchanged:

$$E_r^* \cos\varphi - E_\varphi^* \sin\varphi = k^*\omega, \quad (4)$$

where k^* is the proportionality factor for transition from an electric field intensity E to angular velocity Ω.

The partial differential equation (2′) is reduced to an ordinary differential equation if we take into consideration the fact that it holds only in the space occupied by the field. Its integral is of the form:

$$(r^2 \sin\varphi E_r^*).(r \sin\varphi E_\varphi^*).(rE_\vartheta^*) = A_0^3.$$

Setting $A_0^3 = 0$, we obtain the simplest solutions of the problem stated: for the case of $E_\varphi = 0$ and finite $E_r E_\vartheta$ we obtain a conic field. The field is shaped as a thin layer (with angular thickness φ_0) on the surface of a truncated cone with angle φ^*. The field direction is approximately determined by a logarithmic spiral on the cone surface. In the case of $E_\vartheta = 0$ and E_r, E_φ being finite, the field takes the form of a layer on the funnel-shaped surface beginning at the origin of the coordinates, and the field is directed along the surface generatrixes.

These two solutions together (as they hold in different parts of space) determine the field shape of those vortex formations, properties of which (see below) are the same as those of electron and proton known from experiment. Namely, the cone angle for electron is $\varphi_1^* = \frac{\pi}{2} - \varphi_0$ and for proton $\varphi_2^* = \varphi_0$, where φ_0 means a limiting, physically infinitesimal angle in the field structure.

So, for an electron the field takes the same form as for those vortices that can be often observed in autumn on a dry cold but sunny day in reaped cornfields. On such days nonuniform heating of soil results in upward flows of heated air and under the action of gusty wind these air flows are swirled and form vortex "funnels" which at first drift almost at the ground and slowly rotate. Sometimes it is possible to notice their "spiral" structure; then they stretch up, grow higher than man's height, the rotation inside them becomes quicker and under the action of a side wind they run rather rapidly along a smooth road and finally disappear from sight. Rather considerable steadiness of these "funnels" is due to their weight; heated air going up in a vortex expands it and thus the vortex existence is maintained.

In the case of an electron, the field is symmetric with respect to the plane which passes through the origin and is perpendicular to the axis of rotation; an electron, therefore, consists of two symmetrical halves.

On the basis of the solutions presented above, which quantitatively determine the electron field, we find for low translational velocities of the electron: $\beta_0^2 \ll 1$; electron mass $m_0 = h_0/c_0^2$ $(\omega_1/2\pi) = h_0\nu_1/c_0^2$, rotational momentum $(J_1 \omega_1)_0 = \frac{1}{2}(h_0/2\pi)$, magnetic moment

$$(\mu_{z_1})_0 = \frac{1}{2}\frac{\varepsilon c_0}{\omega_1} = \frac{1}{2}\frac{\varepsilon}{m_0 c_0}\cdot\frac{h_0}{2\pi};$$

gyromagnetic ratio

$$G_1 = \frac{(J_1\omega_1)_0}{(\mu_{z_1})_0} = 1 \times \frac{m_0}{\varepsilon/c_0}.$$

for brevity we use the notation

$$\frac{h_0}{2\pi} = \frac{\varepsilon^2}{c_0}\cdot\frac{4{\cdot}50}{\varphi_0}.$$

From the physical meaning h_0 is the Planck constant. For velocities lower than the velocity of light, i.e. $\beta_0^2 < 1$ we get

$$m = \frac{m_0}{\sqrt{(1 - \beta_0^2 + 2_0^2\varphi_0^2)}}; (J_1\omega_1) = (J_1\omega_1)_0 .$$
$$\times \sqrt{1 - \beta_2^0 + 2\beta_0^2\varphi_0^2)};$$

$$\mu_{z1} = (\mu_{z4})_0 \times (1 - \beta_0^2 + 2\beta_0^2\varphi_0^2).$$

For the case of a proton ($\varphi_2^* = \varphi_0$), the field shape will be similar to that of a vortex field in the case of a storm; it also consists of two symmetrical halves, as it is for an electron; for $\beta^2 \ll 1$ proton mass $M_0 = 2(h_0\nu_2/c_0^2)$, rotational momentum

$$(J_2\omega_2)_0 = 1\frac{h_0}{2\pi}$$

and magnetic moment

$$(\mu_{z_2})_0 = \frac{1}{4}\frac{\varepsilon c_0}{\omega_2} = \frac{1}{2}\frac{\varepsilon}{M_0 c_0}\cdot\frac{h_0}{2\pi};$$

gyromagnetic ratio

$$G_2 = \left(\frac{J_2\omega_2}{\mu_{z_2}}\right)_0 = 2\cdot\frac{M_0}{\varepsilon}\Big/ c_0\,.$$

At velocities lower than light velocity $\beta_0^2 < 1$
1 proton mass

$$M = \frac{M_0}{\sqrt{(1+\beta_0^2 - 2\beta_0^2\varphi_0^2)}};$$

rotational momentum:

$$(J_2\omega_2) = (J_2\omega_2)_0\cdot\sqrt{(1+\beta_0^2 - 2\beta_0^2\varphi_0^2)}$$

and magnetic moment:

$$\mu_{z_2} = (\mu_{z_2})_0\cdot(1+\beta_0^2 - 2\beta_0^2\varphi_0^2).$$

From the expression for m_0 and M_0 we have

$$\frac{m_0}{M_0} = \frac{1}{2}\varphi_0^2,$$

as in general

$$\frac{\omega_1}{\omega_2} = \frac{\cos\varphi_1^*\,.\,\sin\varphi_2^*}{\sin\varphi_1^*\,.\,\cos\varphi_2^*};$$

Since experimentally $m_0/M_0 = 1/1838$, we have $\varphi_0 = 0{\cdot}033$ and from $h_0/2\pi$ we see that $\varphi_0/4{\cdot}50 = \alpha_0 = 1/137$ is a constant of fine structure.

For $\beta_0^2 = 1$, from indirect considerations, these formations (electron and proton) are not stable and cannot exist, they disappear.

When the field of two halves of proton is asymmetric, there appears a neutron with the mass similar to that of proton (but not equal to it exactly, as the energy of an eddy-free field is different for both of them); in the case of asymmetry of the two halves of an electron there appears a formation similar to the latter and with mass equal to that of an electron but neutral. This is neutrino.

The photon structure is more complicated than that of an electron and proton. In the first approximation the third possible case of the general solution when $A_0^3 = 0$ and $E_r \to 0$, E_φ and E_ϑ are finite, gives us some idea of a photon field: this is a field in the form of a layer on the surface of sphere r^*; it is possible on the basis of (3) and (4) only with the translational velocity equal to the light velocity: $\beta_0^2 = 1$. The field direction on the sphere is determined by a thumb line which crosses meridians at an angle of 70°.

This brief description of the main field structure properties for an electron and a proton shows that quantitative relations for an electron are in full agreement with all that is known from experimental theories; this concerns first of all gyromagnetic ratio G which according to Maxwell's theory turns out to be twice as large.

What is quite new is the dependence of rotational momentum and magnetic moment of both electron and proton on their translational velocities. And it is absolutely unexpected that proton mass decreases with increase of its translational velocity.

As for the wave properties of an electron, it is clear that in an eddy-free region the rotating electric field of an electron produces pressure waves and depression-type waves with amplitude proportional to φ_0, and spreading with velocity $v_0 = c_0^2/u_0$, if u_0 is the translational velocity of an electron. We get this main wave relation directly from the system of equations in the second approximation for an eddy-free gas motion. For a plane wave, these equations will be written in the form

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} + \frac{\partial u}{\partial x} = 0;$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) + \frac{\mathrm{d}u}{\mathrm{d}t} = 0;$$

under the condition

$$\frac{\partial\rho}{\mathrm{d}t} + u_0\frac{\partial\rho}{\partial x} = 0.$$

As a common feature of those fields obtained when solving the basic equations of the electromagnetic field in the second approximation, it is necessary to note the absence of singular points.

Comparing the solutions obtained for an electron (positron) and a proton, we may predict that if we manage in experiment with the aid of an external magnetic field to increase the velocity of positron rotation by $1838/2 = 919$ times, it will be converted into a proton, and an electron in a similar way must be converted into anti-proton, which as yet has not been obtained by experimental physicists.